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DEVELOPMENT OF A UNIVERSAL MATHEMATICAL MODEL OF HIGH PRESSURE DISCHARGE LAMPS

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Abstract: A differential-algebraic model of the kit "high pressure discharge lamp - inductive ballast" has been developed by gradual modification. The purpose of the modifications of the basic model of a high pressure mercury discharge lamp was to develop a model of high pressure discharge lamp for different types, provided with a calculation stability for different initial conditions and external perturbations. By computing experiment the behavior of electrical parameters of high pressure discharge lamps has been investigated. Comparison of the obtained results with real data confirms the adequacy of the model.

1. INTRODUCTION

The lighting systems for domestic and industrial applications use different light sources – incandescent lamps, discharge lamps and LEDs [4]. To date, high pressure discharge lamps are considered to be the most optimal ones, with high levels of durability, efficiency and economical operation, with sources of optical radiation, designed for illumination of industrial and civil objects (HPD Lamps) [10]. Their work is provided by ballasts of different principles of action [4, 8, 10]. Although in recent years electronic ballasts are becoming more and more widespread, electromagnetic ones still remain the most common. The simplest electromagnetic ballast is a successively engaged throttle (inductive ballast) (fig. 1).

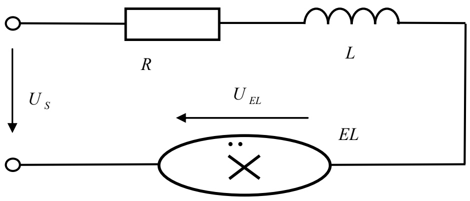

Fig. 1. The scheme of switching on HPDL with inductive ballast

On figure 1 used next symbols:
U_S – voltage sours;
R – active resistor (active power lose in scheme);
L – inductivity of ballast;
EL – High Pressure Discharge Lamp;
U_{EL} – voltage on High Pressure Discharge Lamp.

2. UNIVERSAL MATHEMATICAL MODEL OF HIGH PRESSURE DISCHARGE LAMPS

2.1. Analysis of recent research and publications

Most known mathematical models [8] focus primarily on an adequate representation of static characteristics of HPDL also suited to describe the steady operation mode. These models are of a purely algebraic nature of approximation and, as a rule, are constructed in the class of ordinary or trigonometric polynomials with the estimated coefficients or partial amount of power or trigonometric series. At the same time, sufficiently detailed analysis of the processes and the ignition start is not carried out, though the peculiarities of the dynamics of changes in the electrical parameters of the lamp (voltage, conductivity, current, power) have a significant impact on the performance of the HPDL and ballasts.

The need for more accurate calculations of transient processes taking into account the non-linear dynamic characteristics of structural components and their relationships requires both the creation of refined mathematical models of these processes, and the development of appropriate engineering methods of their application. Thus, there is an actual problem of constructing models of transient HPDL modes allowing their effective use.

2.2. Object of the article

To create a numerically stable modification of the existing model [8] intended to describe transient modes of the kit HPDL-inductive ballast in arbitrary initial conditions using modern computational methods.

2.3. Modification of the Basic Model of the Kit High Pressure Discharge Lamp – Inductive Ballast

Suppose t – time, c; u_S – external power supply, B; L – throttle inductivity, Гн; R – active resistance of the circuit, Ohm; u_{EL} – voltage across the lamp, B; U_0 – rated voltage across the lamp, B; i_{EL} – strength of the current through the lamp, A; P_{EL} – lamp power, W; P_0 – nominal power of the lamp, W; G_{EL} – lamp conductivity, Cm; g_{EL} – powered lamp conductivity reflecting the average concentration of electrons, Cm; β – dimensionless variable proportionality coefficient reflecting the mobility of electrons.

Taken as a basis for differential-algebraic model HPDL [8]:

$$dg_{EL}/dt = \frac{g_{EL}^2 \cdot A_{EL} \cdot U_0^2 \left[\left(u_{EL}/U_0\right)^2 - 1 \right]}{\left[1 + k_1\left(|u_{EL}|/U_0 - 1\right)\right]} \tag{1}$$

$$d\beta/dt = \left[k_2 + k_3\left(|u_{EL}|/U_0\right)^{k_4}\right] \times$$
$$\times\left[1 + k_1\left(|u_{EL}|/U_0 - 1\right) - \beta\right] \quad (2)$$

$$G_{\mathrm{EL}} = \beta g_{EL}; \qquad i_{EL} = G_{EL} u_{\backslash EL} \quad (3)$$

supplemented by the equation

$$di_{EL}/dt = (1/L)\left(u_S - i_{EL}R - i_{EL}/G_{EL}\right) \quad (4)$$

which describes the second Kirchhoff's law for circuit series connection of HPDL with inductive ballast using the active resistance.
Where A_{EL}, k_1, k_2, k_3, k_4 – estimated constant coefficients. In accordance to [3]:

$$k_1 = 0{,}6;\ k_2 = 1{,}5\cdot 10^4;\ k_3 = 3\cdot 10^4;\ k_4 = 1{,}5, \quad (5)$$

and the value of the coefficient A_{EL} depends on the type of lamp. In particular, for DRL-400 lamp [8]:

$$P_0 = 400; \quad U_0 = 131; \quad A_{\mathrm{EL}} = 5{,}5. \quad (6)$$

Thus, the kit "HPDL-inductive ballast" is modeled by DAS (1) - (4), consisting of three differential equations and two algebraic ones.

Provided $G_л \neq 0$ index differentiating [1, 6] DAS (1) – (4) is equal to one, which allows reducing it to an extended ODE system. However, this increases the computational costs when operating with an increased differential dimension model. A more productive approach is based on the exclusion of some of the variables of differential subsystem. Eliminating some of these variables, it is possible to go to a purely differential system without increasing its dimensions. Moreover, since some of the variables (β and g_{EL}) are not observed immediately, it is natural to use the state-space method to describe the object "HPDL-inductive ballast".

From the second constraint equation (3) it is possible to express variable u_{EL} through other variables: $u_{EL} = i_{EL}/(\beta g_{EL})$, and then eliminate u_{EL} and G_{EL} from the original differential subsystem.

As a result, it takes the following form:

$$dg_{EL}/dt = \frac{A_{EL}U_0^2\left[\left(i_{\mathrm{EL}}/(\beta U_0)\right)^2 - g_{EL}^2\right]}{\left[1 + k_1\left(|i_{EL}|/(\beta g_{EL}U_0) - 1\right)\right]} \quad (7)$$

$$d\beta/dt = \left[k_2 + k_3\left(|i_{EL}|/(\beta g_{EL}U_0)\right)^{k_4}\right] \times$$
$$\times\left[1 + k_1\left(|i_{EL}|/(\beta g_{EL}U_0) - 1\right) - \beta\right] \quad (8)$$

$$di_{EL}/dt = (1/L)\left(u_S - i_{EL}R_{nom} - i_{EL}/(\beta g_{EL})\right) \quad (9)$$

To reduce the influence of rounding errors and to achieve proportionality of different variables, it is advisable to go to dimensionless quantities:

$$\bar{t} = \frac{t}{T_0};\ \ x_1 = \frac{g_{EL}}{g_0};\ \ x_2 = \beta;\ \ x_3 = \frac{i_{EL}}{I_0}; \qquad (10)$$

$$y_1 = x_3;\ \ y_2 = \frac{u_{EL}}{U_0} = \frac{x_3}{x_1 x_2};\ \ \ y_3 = \frac{G_л}{G_0} = x_1 x_2; \qquad (11)$$

$$y_4 = \frac{P_{EL}}{P_0} = \frac{u_{EL} i_{EL}}{P_0} = \frac{x_3^2}{x_1 x_2};\ \ w = \frac{u_S}{U_0}, \qquad (12)$$

where $T_0, P_0, U_0, I_0, g_0, G_0$ – characteristic values of relevant electrical parameters. Given U_0 – lamp burning voltage at DC and P_0 – nominal power of the lamp; f and $T_0 = 1/f$ – frequency and period of the external power supply (at power frequency $f = 50$ Hz period $T_0 = 0{,}02$ c); $I_0 = P_0 / U_0$ – typical current; $g_0 = G_0 = P_0 / U_0^2$ – typical conductivity.

After simple transformations the following model of transient modes of the kit "HPDL-*inductive ballast*" can be obtained using state-space (above the dimensionless time $\bar{t}$ the trait is omitted):

$$\begin{cases} dx_1/dt = \dfrac{A_л T_0 P_0 \left[\left(x_3 / x_2 \right)^2 - x_1^2 \right]}{\left[1 + k_1 \left(|x_3| / (x_1 x_2) - 1 \right) \right]}; \\ dx_2/dt = T_0 \left[k_2 + k_3 \left(|x_3| / (x_1 x_2) \right)^{k_4} \right] \times \\ \times \left[1 + k_1 \left(|x_3| / (x_1 x_2) - 1 \right) - x_2 \right]; \\ dx_3/dt = \left(T_0 / (L P_0) \right) \cdot (U_0^2 w - P_0 R_{nom}\, x_3 - \\ -U_0^2\, x_3 / (x_1 x_2); \end{cases} \qquad (13)$$

$$\begin{aligned} &y_1 = x_3;\ \ y_2 = x_3 / (x_1 x_2); \\ &y_3 = x_1 x_2;\ \ y_4 = x_3^2 / (x_1 x_2), \end{aligned} \qquad (14)$$

Where $x_i = x_i(t)$, $i = \overline{1,3}$ – phase variables (state variables); $y_j = y_j(t)$, $j = \overline{1,4}$ – output (observed) variables (current, voltage, conductivity and power of the lamp); $w = w(t)$ – input (disturbing) impact; t – time, $t \in [0;T]$; T – length of time to study the process.

In the calculations of specific processes the considered model (13) - (14) is supplemented by the initial conditions:

$$x_i(0) = x_{i0},\ \ i = \overline{1,3}., \qquad (15)$$

The differential system (13) is substantially non-linear; it comprises the steps of dividing into the phase variables, one of which x_1 can physically take values close to zero, which gives rise to local instability of the system solutions. In other words, the differential system is locally rigid [1, 6].

The rigidity of the system varies over a wide range up to the degeneration of the differential equations into algebraic ones with formally admissible value $x_1 = 0$. Therefore, for the regularization of the problem when dividing into variable expressions x_1 a small parameter $\varepsilon > 0 : x_1 x_2 \to x_1 x_2 + \varepsilon$ is introduced. The value ε in its physical sense is an estimate of the unspecified component of conductivity of the lamp in the starting mode in the initial model. This leads to the presence of small-current glow discharge between the main and auxiliary electrodes. Given $\varepsilon = 10^{-3}$.

Modern and effective methods for solving rigid systems of ordinary differential equations [1, 6] are supposedly used on the right side of the system.

However, the system (13) includes a non-differentiable at zero capture module operation $|x_3|$, therefore a replacement for increasing the stability of computing process is applied $|x_3| \to \sqrt{x_3^2 + \delta}$, where $\delta > 0$ – small positive number. Since the practical calculation of electrical parameters are maintained with an accuracy of no more than six significant digits, it is given $\delta = 10^{-14}$.

Thus, the model of transient modes of the kit "HPDL - inductive ballast" takes the form:

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{A_{\text{EL}}\, T_0 P_0 \left[\left(x_3 / x_2 \right)^2 - x_1^2 \right]}{\left\{ 1 + k_1 \left[\sqrt{x_3^2 + \delta} \Big/ \left(x_1 x_2 + \varepsilon \right) - 1 \right] \right\}}; \\[3ex] \dfrac{dx_2}{dt} = T_0 \left\{ k_2 + k_3 \left[\sqrt{x_3^2 + \delta} \Big/ \left(x_1 x_2 + \varepsilon \right) \right]^{k_4} \right\} \times \\ \times \left\{ 1 + k_1 \left[\sqrt{x_3^2 + \delta} \Big/ \left(x_1 x_2 + \varepsilon \right) - 1 \right] - x_2 \right\}; \\[3ex] \dfrac{dx_3}{dt} = \dfrac{T_0}{L P_0} [U_0^2 w - P_0 R x_3 - U_0^2\, x_3 \Big/ \left(x_1 x_2 + \varepsilon \right)]; \end{cases} \tag{16}$$

$$\begin{gathered} y_1 = x_3;\; y_2 = x_3 / (x_1 x_2 + \varepsilon); \\ y_3 = x_1 x_2;\; y_4 = x_3^2 \Big/ (x_1 x_2 + \varepsilon), \end{gathered} \tag{17}$$

The modified system of ODE (16) is presented in normal form, the right-hand side of which in arbitrary physically acceptable initial conditions is continuous in all arguments.

With the continuity of the input action $w = w(t)$ the right side satisfies in the variables $x_i = x_i(t)$, $i = \overline{1,3}$ Lipschitz condition with some positive constant M. Therefore, according to Cauchy's theorem, the initial value problem (15), (16) has a solution. The degree of stability of the numerical solution is determined by the Lipschitz constant M that is adjusted by the choice of regularization parameter ε and δ.

The requirement for economical use of computing resources in a system of variable rigidity leads to choices for solving the Cauchy problem of special numerical methods that allow changing the order of accuracy and the integration step quite easily.

In today's software environments with advanced system of mathematical computations such as MATLAB [7], there are solvers that meet the requirements. Among them, the most efficient are currently recognized solvers, in particular, ode15s [3], implementing the implicit multivalued Gere's method in Nordsieck's introduction [1, 6].

2.4. Implementation of the Model of the Kit High Pressure Discharge Lamp – Inductive Ballast

Mathematical model of the kit "HPDL - *inductive ballast*" is implemented in MATLAB software environment [3]. This package contains an effective unit of applied mathematical calculations for various purposes, including solution of the Cauchy problem (3) - (5), as well as a means of synthesis of the input signals of different shapes and developed ways to analyze and visualize the output data.

Given the existence of rigidity in the differential system, solver ode15s has been selected for solving the Cauchy problem, which implements an implicit multivalued Gere's method to automatically change the accuracy order and integration step [2].

This makes it possible to produce stable calculations with limited cost of computing resources.

According to the accepted guidelines [3, 5], the corresponding m-files are created in the MATLAB environment. In the file-function RDSOE1.m the parameters of the kit HPDS-inductive ballast are set, as well as the system of differential equations, the nature of the supply voltage and auxiliary calculation formula.

In the file-script RDscr1.m a request is made to the solution of the Cauchy problem with an indication of the initial conditions and the interval of integration, and also the file contains commands for calculating output variables and output results.

2.5. Comparison of the Results of Numerical Experiments With Real Data.

Fig. 2 show the results of the simulation of the kit HPDL - inductive ballast in a graphical form.

The calculations used characteristics of the lamp DRL-250. Given: throttle inductivity $L = 0{,}175$ Gn; loss resistance in the circuit $R = 5$ Ohm; rated lamp power $P_0 = 250$ W; the nominal value of the lamp burning with voltage $U_0 = 126$ DC V. The model coefficients $A_0 = 9$, $k_1 = 0{,}6$, $k_2 = 1{,}5 \cdot 10^4$, $k_3 = 3 \cdot 10^4$, $k_4 = 1{,}5$.

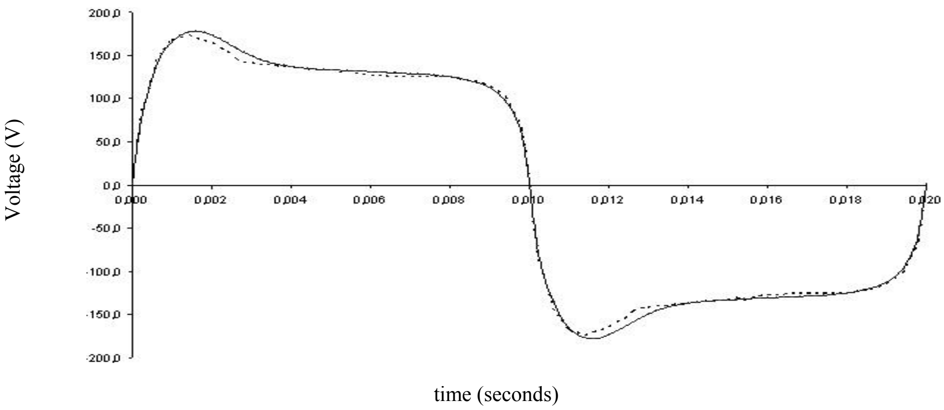

Fig. 2. Plots of the lamp voltage: dotted line - experimental data; solid line - model values.

The interval of integration $[0;T]$, where $T = 30$ intervals (i.e. finite time at mains frequency $f = 50$ Hz is $T = 0,6$ s). The initial conditions given: $x_1(0) = 0,005$ и $x_2(0) = 0,4$ (i.e. initial conductivity of the lamp $G_{EL}(0) = x_1(0) \cdot x_2(0) \cdot P_0 / U_0^2$ then $G_{EL}(0) = 4,66 \cdot 10^{-5}$ Cm); $x_3(0) = 0$ (i.e. initial current of the lamp $i_{EL}(0) = x_3(0) \cdot P_0 / U_0 = 0$ A). Input action is a sinusoidal voltage of the mains frequency $w = (220 / U_0)\sqrt{2}\sin(2\pi t)$, $0 \le t \le 30$ ($U_S = 220 * \sqrt{2} * \sin(100\pi t)$ B). According to the diagrams (fig 2 -. 5) the steady state comes after T = 24, that is 0.48 c.

It should be noted that the nonsinusoidality of the processes is clearly demonstrated in Fig. 2. In such a case, the influence of the initial conditions is manifested in different duration of transients before reaching the steady state. Comparing the model values of the lamp voltage $u_л$ (within one interval of the steady state) with the experimental data is carried out by combining the graphs in Fig. 2.

Analysis of the values and forms of these graphs proves the acceptable quality of the model. According to the calculations, the mean square error of determination $u_л$ is 4.7%.

3. CONCLUSIONS

In contrast to the basic model (1) - (4), the suggested modification (16) - (17) has been introduced for the first time in work [7]. It ensures the existence and uniqueness of the solution in arbitrary initial conditions. Choosing an effective numerical method for its software implementation allows for sustainable settlements with limited cost of computing resources.

This modification is the basis for the creation of research methodology of transient modes of the kit HPDL – inductive ballast by means of simulation tools in MATLAB software environment with SIMULINK subsystem.

Analysis of simulation results as well as their direct comparison with the data of field experiments show that the developed model (16), (17) adequately describes the modes of the kit HPDL – inductive ballast. This is also confirmed by the consistency of the results with theoretical calculations [8, 9]. To reduce the modeling error it is supposed to conduct parametric identification of the model used in relation to kits with different types of HPDL.

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